An ERC Starting grant to study unstable dynamics

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European Research Council
Established by the European Commission

The ERC program

- Individual researcher grants awarded by the European Research Council.
- Large funding to set up a research team.
- High-risk, high-reward proposals.
- Three different types:
    - Starting: Up to 7 years after the PhD
    - Consolidator: From 7 to 12 years after the PhD
    - Advanced: More than 10 years after the PhD

The panels for Physical sciences and Engineering

- PE1 Mathematics.
- PE2 Fundamental Constituents of Matter.
- PE3 Condensed Matter Physics.
- PE4 Physical and Analytical Chemical Sciences
- PE5 Synthetic Chemistry and Materials
- PE6 Computer Science and Informatics
- PE7 Systems and Communication Engineering
- PE8 Products and Processes Engineering
- PE9 Universe Sciences
- PE10 Earth System Sciences

The ERC Starting grant

- Who can apply: Researchers of any nationality with 2-7 years of experience since completion of PhD.
- Funding: Up to 1.5 million euros for a period of 5 years.
- Allows to pay the salary of the PI.
- Application:
  - Submission of a proposal (with a thorough referee process).
  - Interview in Brussels with the panel.
- At UPC: Support from CTT – Oficina de projectes europeus (Esther Alsina, Montse Banegas).
- Heavy bureaucracy: Support from UTG – ETSEIB.

The 2017 call

- 406 ERC Starting grant awarded.
- Success rate: 13%.
- 12 in the panel of Mathematics.
- My proposal: Instabilities and homoclinic phenomena in Hamiltonian systems.
- Within the dynamical systems group at UPC (ETSEIB).

My research

Study unstable behavior in different dynamical systems

- Dynamical systems: models of phenomena which evolve in time (differential equations).
- Start with a simple model, easy to analyze.
- What happens if we slightly modify it?

  (We were omitting small forces, we want a more precise model)
- Question: Does the change in the model create effects accumulating over time and leading to instability or it averages out (stability)?

The models

- The N-body problem: N punctual bodies moving under the influence of the Newtonian gravitational force.

  Classical (most simple) model of the Solar system.

- Hamiltonian Partial Differential Equations: Partial Differential Equations which are Hamiltonian systems in infinite dimensional spaces.

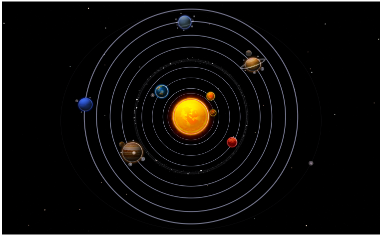
  Examples: Nonlinear Schrödinger equation, Nonlinear wave equation, Euler equation, Water waves equation,...

- Disclaimer: To prove theorems we need simple models!

The N body problem

$$\frac{d^2 q_i}{dt^2} = \sum_{j=1, j\neq i}^{N} m_j \frac{q_j - q_i}{\|q_j - q_i\|^3}, \qquad q_i \in \mathbb{R}^3, i = 1 \dots N.$$

- Solar system: N body problem with one massive body and many small bodies.
- Kepler laws: Planets move on ellipses.
- Only considers Sun - Planet interaction!!



- We should include the (much smaller) Planet–Planet interactions.

Is the Solar system stable or not?

- What happens if we include Planet-Planet interactions?
- Do still the planets perform orbits "similar" to ellipses (stability) or do they deviate a lot if we wait time enough (instability)?
- Numerics (Laskar): Measurement errors of 15 meters of the position of the earth makes leads to unpredictable future position after ~ 100 milion years.
- Goal: Understand mechanisms leading to unstable orbits.

The nonlinear Schrödinger equation

$$-i\partial_t u + \nabla^2 u = |u|^2 u \quad \text{(with periodic boundary conditions)}$$

- For small solutions, this equation is close to the linear one
- The linear equation is easy to solve: Fourier series.
- Which new dynamics can create the non-linear term?
- Unstable behavior? (far from the linear equation behavior)

The nonlinear Schrödinger equation

$$-i\partial_t u + \nabla^2 u = |u|^2 u \quad \text{(with periodic boundary conditions)}$$

- Transfer of energy: solutions go from oscillating at low frequencies to oscillating higher and higher frequencies as time evolves.
- Fundamental problem in many settings (fluid mechanics).
- Related with turbulence theory (weak turbulence).